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46. Proposed by H. C. WHITAKER, M. E., Sc. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

There are four points A, B, C, and D in space. Point D remains fixed with its co-ordinates (1, 2, 2) feet. At a given time A is at (2, 3, 4) feet, is moving in a straight line at the rate of 3 feet per minute, and has passed through (5, 9, 10) feet; B is at (1, 4, 2) feet, moves in a straight line at the rate of 7 feet per minute, and will pass through (—2, 2, 8) feet; C is at the origin and moves along the axis of X in the direction of x positive at the rate of 6 feet per minute.

The motion of the points being continuous before and after the given time, required the times when the volume of the tetrahedron whose edges are the lines joining these points will be 108 cubic inches.

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MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

26. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

If an elastic sphere be electrified in such a manner that the initial internal pressure remains constant, determine an expression for the *ratio of the electrical densities* when the volume of the sphere has been increased to $(m+1)$ times its initial volume.

I. Solution by the PROPOSER.

Assuming r as the *initial* radius, and R as any *greater* radius, we have $x^2+y^2+z^2=r^2$ ....(1) and $x^2+y^2+z^2=R^2$ ....(2), as the result of the *volume-increase*. These Cartesian equations are transformable by means of the well-known equations, $x=\rho\cos\theta$ and $y=\rho\sin\theta$. Choosing the integral limits so as to give the total internal pressure, we have for the work done in increasing the initial sphere, radius r, to a sphere of radius R, when $z_1=\sqrt{(r^2-\rho^2)}$ and $z_2=\sqrt{(R^2-\rho^2)}$,

$$W=8\left(\frac{2T}{r}\right)\int_0^{\frac{1}{2}\pi}\int_0^r 2\rho d\theta d\rho\times 8\int_0^{\frac{1}{2}\pi}\left[\int_0^R\int_0^{z_2}\rho d\rho dz-\int_0^r\int_0^{z_1}\rho d\rho dz\right]d\phi$$

$$=\tfrac{4}{3}\pi(8\pi Tr)[R^3-r^3]\text{....(3).}$$

Specializing in (3), for $R^3=mr^3$ and for $R^3=(m+1)r^3$; representing the electrical densities, the ratio of which is to be determined, by Δ_m and Δ_{m+1}; then equating the specialized results obtained from (3), to the potential energies of the electrifications similarly specialized,—that is, according to Helmholtz's formula, to

$$\frac{Q}{2r}=\left(\frac{\Delta_n^2}{2r}\right)\left[8\int_0^{\frac{1}{2}\pi}\int_0^r 2\rho d\theta d\rho\right]^2,$$

we have the required ratio,

$$\frac{\Delta_m}{\Delta_{m+1}}=\sqrt{\left(1-\frac{1}{m^2}\right)}\ldots.(4).$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in the Texarkana College, Texarkana, Arkansas.

Let p be the excess of pressure, on the assumption that the external pressure is costant.

Let u=original volume, then when the sphere is n times its original volume, the work done in this electrification must be

$p(n-1)u$, (see *Minchin's Statics*).

Let v be the potential, Q the charge of this electrification, r=radius of sphere, σ=electrical density.

$\therefore$ energy of electrification$=\frac{1}{2}vQ$. But $v=\frac{Q}{r}$ and. $Q=4\pi r^2\sigma$.

$\therefore p(n-1)u=\frac{1}{2}\frac{Q^2}{r}=8\pi^2 r^3\sigma^2$. But $\frac{4}{3}\pi r^3=nu$.

$\therefore p(n-1)u=6n\pi\sigma^2 u$.

$\therefore p(n-1)=6n\pi\sigma^2$.

Similarly, when the sphere becomes $(m+1)$ times its original size we get, if σ_1 is the density, $pm=6(m+1)\pi\sigma_1^2$.

$$\therefore \frac{\sigma}{\sigma_1}=\sqrt{\frac{(m+1)(n-1)}{mn}}=\sqrt{\frac{m^2-1}{m^2}} \text{ if } n=m.$$

27. Proposed by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in the Texarkana College, Texarkana, Arkansas.

One thousand balls, each having a mass of 10 grams, and each moving with a velocity of 10 kilometers per second, are confined in a certain space with elastic walls. Into the same space are now introduced one thousand balls each of 1000 grams mass, and moving with a velocity each of 10 kilometers per second; collisions take place, and finally, after a number of encounters, the average kinetic energy of each of the two thousand balls is the same. Show that this is 5.75(10)11 in the centimeter-gram-system.

Solution by O. W. ANTHONY, Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland and the PROPOSER.

By Avogadro's hypothesis and kinetic theory, if M, M_1 are the masses and V, V_1 the velocities of two balls, then $\frac{1}{2}MV^2$ and $\frac{1}{2}M_1V_1^2$ are their respective kinetic energies. As the average kinetic energy of each ball is equal, we get $\frac{1}{2}MV^2=\frac{1}{2}M_1V_1^2$.

$\therefore$ The average kinetic energy$=\frac{1}{2}(\frac{1}{2}MV^2+\frac{1}{2}M_1V_1^2)=E$.

$\therefore E=\frac{1}{4}(MV^2+M_1V_1^2)$.

Let $V=V_1$, then $E=\frac{1}{4}(M+M_1)V^2$.

V=10 kilometers$=10^6$ centimeters.

M=10 grams, M_1=100 grams.

$\therefore E=\frac{1}{4}(10+100)10^{12}=\frac{11}{4}\times10^{13}=2.75\times10^{13}=2.75(10)^{13}$.

Also solved by *F. P. MATZ.*